Model–independent analysis of the exclusive semileptonic $B \to \pi^+ \pi^- \ell^+ \ell^-$ decay

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Abstract

A model–independent analysis for the exclusive, rare $B \to \pi^+ \pi^- \ell^+ \ell^-$ decay is presented. The dependence of the various physically measurable asymmetries and CP violating asymmetry on the new Wilson coefficients is studied in detail. It is observed that different measurable quantiles are very sensitive to the different new Wilson coefficients, i.e., to the existence of new physics beyond standard model.

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1 Introduction

Rare B meson decays, induced by flavor–changing neutral current (FCNC) $b \to s$ or $b \to d$ transitions, occur at loop level in the standard model (SM) and is a potential precision testing ground for the SM, as well as attracting the theoretical interest as a promising tool for establishing new physics beyond SM. New physics effects show themselves in rare B meson decays in two different ways, namely, through the Wilson coefficients which could be distinctly different from their SM counterparts or through the new structures in the effective Hamiltonian (see for example [1]– [13]). One of the main goal of the working B factories and of the future hadron colliders is the study of the rare B meson decays. Therefore theoretical and experimental investigation of the rare decays of B mesons receive special attention. The aim of this paper to investigate the rare decays $\bar{B} \to K^-\pi^+\ell^+\ell^-$ and $\bar{B} \to \pi^+\pi^-\ell^+\ell^-$ when $K\pi$ and $\pi\pi$ systems are the decay products of K^* and ρ mesons, in a model–independent manner.

The inclusive $b \to s\ell^+\ell^-$ and exclusive $B \to K^*\ell^+\ell^-$ decays were analyzed in a model–independent way in [11] and [12], respectively. The present work is an extension of the previous studies of the $B \to K^*(\rho)\ell^+\ell^-$ decay. The distribution in $\cos\theta_P$, where θ_P is the angle of the $K^-(\pi^-)$ in the $K^-\pi^+(\pi^-\pi^+)$ center of mass frame, and the dependence on the azimuthal angle φ between the $\ell^+\ell^-$ and $K^-\pi^+$ or $\pi^-\pi^+$ planes, which does not exist in the $B \to K^*(\rho)\ell^+\ell^-$ decay, can provide additional new information. This information is sensitive to the polarization state of the vector meson $K^*(\rho)$ and therefore opens new possibility in probing the structure of the effective Hamiltonian. Angular distributions and CP asymmetries in the $\bar{B} \to K^-\pi^+\ell^+\ell^-$ and $\bar{B} \to \pi^+\pi^-\ell^+\ell^-$ decays in the SM were studied in [14].

The paper is organized as follows. In section 2, we present the general, model independent expression of the decay distribution for the $B \to K\pi(\pi\pi)\ell^+\ell^-$ decay, in terms of the helicity amplitudes, including the non–zero lepton mass effects. In section 3, we present our numerical analysis for different angular distribution together with a brief concluding remark of our results.

2 Theoretical background

The matrix element for the $B \to K^*(\rho)\ell^+\ell^-$ decay at quark level is described by the $b \to f\ell^+\ell^-$ $(f = s, d)$ transition. Following the works [10]–[12], the matrix elements of the $b \to f\ell^+\ell^-$ decay can be written as the sum of the SM and new physics contributions as

$$
\begin{aligned}
\mathcal{M}_1 &= \frac{G\alpha}{\sqrt{2}\pi} V_{tb}V_{tf}^* \Bigg\{ C_{SL}\, \bar{f} i\sigma_{\mu\nu} \frac{q^\nu}{q^2}\, L\, b\, \bar{\ell}\gamma_\mu \ell + C_{BR}\, \bar{f} i\sigma_{\mu\nu} \frac{q^\nu}{q^2}\, R\, b\, \bar{\ell}\gamma_\mu \ell \\
&+ C_{LL}^{tot}\, \bar{f}_L\gamma_\mu b_L\, \bar{\ell}_L\gamma^\mu \ell_L + C_{LR}^{tot}\, \bar{f}_L\gamma_\mu b_L\, \bar{\ell}_R\gamma^\mu \ell_R + C_{RL}\, \bar{f}_R\gamma_\mu b_R\, \bar{\ell}_L\gamma^\mu \ell_L \\
&+ C_{RR}\, \bar{f}_R\gamma_\mu b_R\, \bar{\ell}_R\gamma^\mu \ell_R + C_{LRLR}\, \bar{f}_L b_R\, \bar{\ell}_L \ell_R + C_{RLLR}\, \bar{f}_R b_L\, \bar{\ell}_L \ell_R \\
&+ C_{LRRL}\, \bar{f}_L b_R\, \bar{\ell}_R \ell_L + C_{RLRL}\, \bar{f}_R b_L\, \bar{\ell}_R \ell_L + C_T\, \bar{f}\sigma_{\mu\nu} b\, \bar{\ell}\sigma^{\mu\nu}\ell \\
&+ iC_{TE}\, \epsilon^{\mu\nu\alpha\beta} \bar{f}\sigma_{\mu\nu} b\, \bar{\ell}\sigma_{\alpha\beta}\ell \Bigg\}~, \qquad (1)
\end{aligned}
$$

the first four terms contain the SM contribution too, while all the others describe the new physics effects and where

$$L(R) = \frac{1-\gamma_5}{2} \ \left(\frac{1+\gamma_5}{2}\right) \ .$$

In further analysis we will neglect the tensor type interactions (terms $\sim \ C_T$ and C_{TE} since physically measurable quantities are not sensitive to the presence of tensor type interactions, as is shown in [10, 11]).

The matrix elements for the precess $B \to PP'\ell^+\ell^-$ can be obtained from the matrix element $B \to V\ell^+\ell^-$ $(V = K^*, \ \rho)$ in the following way

$$\mathcal{M} = \mathcal{M}_{B\to V\ell^+\ell^-} \, \Pi(s_M) \langle \pi^+(p_+)\pi^-(p_-)|V(p_V,\lambda)\rangle \ , \tag{2}$$

where we assume that the resonance contribution of the intermediate vector meson can be implemented by the Breit–Wigner form

$$\Pi(s_M) = \frac{\sqrt{m_V \Gamma_V/\pi}}{s_M - m_V^2 + i m_V \Gamma_V} \ , \tag{3}$$

where $s_M = (p_+ + p_-)^2$ and m_V and Γ_V are the mass and width of the vector meson $(K^*, \ \rho)$. For the decay part of the vector meson we use [15]

$$\langle P(p_+)P'(p_-)|V(p_V,\lambda)\rangle = \sqrt{BR} \; Y_{\lambda_{max}}^{\lambda_i}(\theta_P,\varphi) \ , \tag{4}$$

where $Y_\ell^m(\theta_P,\varphi)$ are the $J=\ell$ spherical harmonics, and the angles θ_P and φ belong to those of the final P meson in the vector meson's rest frame. The coupling of $V \to PP'$ decay is effectively taken into account by the branching ratios.

From (2) we see that in order to calculate the matrix element $B \to PP'\ell^+\ell^-$, the matrix element of the $B \to V\ell^+\ell^-$ decay is needed in the first hand, for which the matrix elements

$$\left\langle V \left| \bar f \gamma_\mu (1 \pm \gamma_5) b \right| B(p_B) \right\rangle \ , \ \left\langle V \left| \bar f i \sigma_{\mu\nu} (1 + \gamma_5) b \right| B(p_B) \right\rangle \ \text{ and } \ \left\langle V \left| \bar f (1 \pm \gamma_5) b \right| B(p_B) \right\rangle \ , \tag{5}$$

need to be calculated, whose most general forms can be written as

$$\begin{aligned}
&\left\langle V(p_V,\varepsilon) \left| \bar f \gamma_\mu (1 \pm \gamma_5) b \right| B(p_B) \right\rangle = \\
&\quad -\epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} p_V^\lambda q^\sigma \frac{2V(q^2)}{m_B + m_V} \pm i \varepsilon_\mu^* (m_B + m_V) A_1(q^2) \mp i (p_B + p_V)_\mu (\varepsilon^* q) \frac{A_2(q^2)}{m_B + m_V} \\
&\quad \mp i q_\mu \frac{2m_V}{q^2} (\varepsilon^* q) \left[A_3(q^2) - A_0(q^2) \right] \ ,
\end{aligned} \tag{6}$$

$$\begin{aligned}
&\left\langle V(p_V,\varepsilon) \left| \bar f i \sigma_{\mu\nu} q^\nu (1 \pm \gamma_5) b \right| B(p_B) \right\rangle = \\
&\quad 4 \epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} p_V^\lambda q^\sigma T_1(q^2) \pm 2i \left[\varepsilon_\mu^* (m_B^2 - s_M) - (p_B + p_V)_\mu (\varepsilon^* q) \right] T_2(q^2) \\
&\quad \pm 2i (\varepsilon^* q) \left[q_\mu - (p_B + p_V)_\mu \frac{q^2}{m_B^2 - s_M} \right] T_3(q^2) \ ,
\end{aligned} \tag{7}$$

where $q = p_B - p_V$ is the momentum transfer and ε is the polarization vector of the vector meson. In order to ensure finiteness of (6) at $q^2 = 0$, we demand that $A_3(q^2 = 0) = A_0(q^2 =$

0). The matrix element $\left\langle V\left|\bar{f}(1\pm\gamma_5)b\right|B\right\rangle$ can easily be calculated from Eq. (6). For this aim it is enough to contract both sides of Eq. (6) with q_μ and use equation of motion. Taking mass of the strange quark to be zero, it gives

$$\begin{aligned}\left\langle V(p_V,\varepsilon)\left|\bar{f}(1\pm\gamma_5)b\right|B(p_B)\right\rangle &= \frac{1}{m_b}\Big\{\mp i(\varepsilon^* q)(m_B+m_V)A_1(q^2)\\ &\pm i(m_B^2-s_M)(\varepsilon^* q)\frac{A_2(q^2)}{m_B+m_V}\pm 2im_V(\varepsilon^* q)\left[A_3(q^2)-A_0(q^2)\right]\Big\}~. \qquad (8)\end{aligned}$$

Furthermore, using the equation of motion, the form factor A_3 can be expressed in terms of the form factors A_1 and A_2 (see [16])

$$A_3(q^2) = \frac{m_B+m_V}{2m_V}A_1(q^2)-\frac{m_B-m_V}{2m_V}A_2(q^2)~. \qquad (9)$$

Using this relation, the matrix element (8) can be written in the following form

$$\left\langle V(p_V,\varepsilon)\left|\bar{f}(1\pm\gamma_5)b\right|B(p_B)\right\rangle = \frac{1}{m_b}\Big\{\mp 2im_V(\varepsilon^* q)A_0(q^2)\Big\}~. \qquad (10)$$

As a result of the above considerations, the matrix element of the $B\to V\ell^+\ell^-$ decay can be determined straightforwardly

$$\begin{aligned}\mathcal{M}(B\to V\ell^+\ell^-) &= \frac{G\alpha}{4\sqrt{2}\pi}V_{tb}V_{tf}^*\\ &\times\Bigg\{\bar{\ell}\gamma_\mu(1-\gamma_5)\ell\left[-2\mathcal{V}_{L_1}\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p_V^V q^\sigma - i\mathcal{V}_{L_2}\varepsilon_\mu^* + i\mathcal{V}_{L_3}(\varepsilon^* q)(p_B+p_V)_\mu + i\mathcal{V}_{L_4}(\varepsilon^* q)q_\mu\right]\\ &+\bar{\ell}\gamma_\mu(1+\gamma_5)\ell\left[-2\mathcal{V}_{R_1}\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p_V^\rho q^\sigma - i\mathcal{V}_{R_2}\varepsilon_\mu^* + i\mathcal{V}_{R_3}(\varepsilon^* q)(p_B+p_V)_\mu + i\mathcal{V}_{R_4}(\varepsilon^* q)q_\mu\right]\\ &+\bar{\ell}(1-\gamma_5)\ell\Big[i\mathcal{S}_L(\varepsilon^* q)\Big]+\bar{\ell}(1+\gamma_5)\ell\Big[i\mathcal{S}_R(\varepsilon^* q)\Big]\Bigg\}~, \qquad (11)\end{aligned}$$

where $\mathcal{V}_{L_i}$ and $\mathcal{V}_{R_i}$ are the coefficients of left– and right–handed leptonic currents with vector structure, and $\mathcal{S}_{L,R}$ are the weights of scalar leptonic currents with respective chirality, respectively, whose explicit forms are given as

$$\begin{aligned}
\mathcal{V}_{L_1} &= (C_{LL}^{tot}+C_{RL})\frac{V(q^2)}{m_B+m_V}-2(C_{BR}+C_{SL})\frac{T_1}{q^2}~,\\
\mathcal{V}_{L_2} &= (C_{LL}^{tot}-C_{RL})(m_B+m_V)A_1-2(C_{BR}-C_{SL})\frac{T_2}{q^2}(m_B^2-s_M)~,\\
\mathcal{V}_{L_3} &= \frac{C_{LL}^{tot}-C_{RL}}{m_B+m_V}A_2-2(C_{BR}-C_{SL})\frac{1}{q^2}\left[T_2+\frac{q^2}{m_B^2-s_M}T_3\right]~,\\
\mathcal{V}_{L_4} &= (C_{LL}^{tot}-C_{RL})\frac{2m_V}{q^2}(A_3-A_0)+2(C_{BR}-C_{SL})\frac{T_3}{q^2}~,\\
\mathcal{V}_{R_1} &= \mathcal{V}_{L_1}(C_{LL}^{tot}\to C_{LR}^{tot}~,~~C_{RL}\to C_{RR})~,\\
\mathcal{V}_{R_2} &= \mathcal{V}_{L_2}(C_{LL}^{tot}\to C_{LR}^{tot}~,~~C_{RL}\to C_{RR})~,\\
\mathcal{V}_{R_3} &= \mathcal{V}_{L_3}(C_{LL}^{tot}\to C_{LR}^{tot}~,~~C_{RL}\to C_{RR})~,\\
\mathcal{V}_{R_4} &= \mathcal{V}_{L_4}(C_{LL}^{tot}\to C_{LR}^{tot}~,~~C_{RL}\to C_{RR})~,\\
\mathcal{S}_L &= -(C_{LRRL}-C_{RLRL})\left(\frac{2m_V}{m_b}A_0\right)~,\\
\mathcal{S}_R &= -(C_{LRLR}-C_{RLLR})\left(\frac{2m_V}{m_b}A_0\right)~,
\end{aligned}$$

In order to obtain the full helicity amplitude of the $B \to PP'\ell^+\ell^-$ which follows from Eq. (2), the helicity amplitude of the $B \to V\ell^+\ell^-$ decay must be written, which we denote as $\mathcal{M}_{\lambda}^{\lambda_\ell\bar{\lambda}_\ell}$

$$\mathcal{M}_{\lambda_i}^{\lambda_\ell\bar{\lambda}_\ell} = \sum_{\lambda_{V^*}} \eta_{\lambda_{V^*}} L_{\lambda_{V^*}}^{\lambda_\ell\bar{\lambda}_\ell} H_{\lambda_{V^*}}^{\lambda_i} \ , \tag{12}$$

where

$$\begin{aligned} L_{\lambda_{V^*}}^{\lambda_\ell\bar{\lambda}_\ell} &= \varepsilon_{V^*}^{\mu} \left\langle \ell^-(p_\ell,\lambda_\ell)\ell^+(p_\ell,\bar{\lambda}_\ell) \left| J_\mu^\ell \right| 0 \right\rangle \ , \\ H_{\lambda_{V^*}}^{\lambda_i\bar{\lambda}_i} &= \varepsilon_{V^*}^{\mu} \left\langle V(p_V,\lambda_i) \left| J_\mu^i \right| B(p_B) \right\rangle \ , \end{aligned} \tag{13}$$

where ε_{V^*} is the polarization vector of the virtual intermediate vector boson satisfying the relation

$$-g^{\mu\nu} = \sum_{\lambda_{V^*}} \eta_{\lambda_{V^*}} \varepsilon_{\lambda_{V^*}}^{\mu} \varepsilon_{\lambda_{V^*}}^{*\nu} \ ,$$

where the summation is over the helicities $\lambda_{V^*} = \pm1, 0, s$ of the virtual intermediate vector meson, with the metric defined as $\eta_\pm = \eta_0 = -\eta_s = 1$ (for more detail see [17, 18]). In Eq. (13), J_μ^ℓ and J_μ^i are the leptonic and hadronic currents, respectively. Using Eqs. (11)–(13) for the helicity amplitudes $\mathcal{M}_{\lambda_i}^{\lambda_\ell\bar{\lambda}_\ell}$ we get the following expressions

$$\begin{aligned} \mathcal{M}_\pm^{++} &= \sin\theta A_\pm^{++} \ , \\ \mathcal{M}_\pm^{+-} &= (-1 \pm \cos\theta) A_\pm^{+-} \ , \\ \mathcal{M}_\pm^{-+} &= (1 \pm \cos\theta) A_\pm^{-+} \ , \\ \mathcal{M}_\pm^{--} &= \sin\theta A_\pm^{--} \ , \\ \mathcal{M}_0^{++} &= \cos\theta A_0^{++} + B_0^{++} \ , \\ \mathcal{M}_0^{+-} &= \sin\theta A_0^{+-} \ , \\ \mathcal{M}_0^{-+} &= \sin\theta A_0^{-+} \ , \\ \mathcal{M}_0^{--} &= \cos\theta A_0^{--} + B_0^{--} \ , \end{aligned} \tag{14}$$

where

$$
\begin{aligned}
A_{\pm}^{++} &= \pm\sqrt{2}m_\ell\Big\{(C_{LL}^{tot}+C_{LR}^{tot})H_\pm + \frac{2}{q^2}\,(C_{BR}G_\pm + C_{SL}g_\pm) + (C_{RR}+C_{RL})h_\pm\Big\}\ ,\\
A_{\pm}^{--} &= -A_{\pm}^{++}\ ,\\
A_{\pm}^{+-} &= \sqrt{\frac{q^2}{2}}\Big\{\Big[C_{LL}^{tot}(1-v)+C_{LR}^{tot}(1+v)\Big]H_\pm + \Big[C_{RL}(1-v)+C_{RR}(1+v)\Big]h_\pm\\
&\quad + \frac{2}{q^2}\,(C_{BR}G_\pm + C_{SL}g_\pm)\Big\}\ ,\\
A_{\pm}^{-+} &= A_{\pm}^{+-}(v\to -v)\ ,\\
A_{0}^{++} &= 2m_\ell\Big[(C_{LL}^{tot}+C_{LR}^{tot})H_0 + (C_{RL}+C_{RR})h_0 + \frac{2}{q^2}\,(C_{BR}G_0 + C_{SL}g_0)\Big]\ ,\\
A_{0}^{--} &= -A_{0}^{++}\ ,\\
B_{0}^{++} &= -2m_\ell\Big\{(C_{LR}^{tot}-C_{LL}^{tot})H_S^0 + (C_{RR}-C_{RL})h_S^0\Big\}\\
&\quad - \frac{2}{m_b}q^2\Big[(1-v)(C_{LRLR}-C_{RLLR}) - (1+v)(C_{LRRL}-C_{RLRL})\Big]H_S^0\Big\}\ ,\\
B_{0}^{--} &= B_{0}^{++}(v\to -v)\ ,\\
A_{0}^{+-} &= -\sqrt{q^2}\Big\{\Big[C_{LL}^{tot}(1-v)+C_{RR}(1+v)\Big]H_0 + \Big[C_{RL}(1-v)+C_{RR}(1+v)\Big]h_0\\
&\quad + \frac{2}{q^2}\,(C_{BR}G_0 + C_{SL}g_0)\Big\}\ ,\\
A_{0}^{-+} &= A_{0}^{+-}(v\to -v)\ ,
\end{aligned}
\tag{15}
$$

where superscripts denote helicities of the lepton and antilepton and subscripts correspond to the helicity of the vector meson (in our case ρ or K^* meson), respectively.

$$
\begin{aligned}
H_\pm &= \pm\lambda^{1/2}(m_B^2,s_M,q^2)\frac{V(q^2)}{m_B+m_V} + (m_B+m_V)A_1(q^2)\ ,\\
H_0 &= \frac{1}{2\sqrt{s_M q^2}}\Bigg[-(m_B^2-s_M-q^2)(m_B+m_V)A_1(q^2)\\
&\quad + \lambda(m_B^2,s_M,q^2)\frac{A_2(q^2)}{m_B+m_V}\Bigg]\ ,\\
H_S^0 &= \frac{\lambda^{1/2}(m_B^2,s_M,q^2)}{2\sqrt{s_M q^2}}\Bigg[-(m_B+m_V)A_1(q^2) + \frac{A_2(q^2)}{m_B+m_V}(m_B^2-s_M)\\
&\quad + 2\sqrt{s_M}(A_3-A_0)\Bigg]\ ,\\
G_\pm &= -2\Big[\pm\lambda^{1/2}(m_B^2,s_M,q^2)T_1(q^2) + (m_B^2-s_M)T_2(q^2)\Big]\ ,\\
G_0 &= \frac{1}{\sqrt{s_M q^2}}\Bigg[(m_B^2-s_M)(m_B^2-s_M-q^2)T_2(q^2) - \lambda(m_B^2,s_M,q^2)\Bigg(T_2(q^2)\\
&\quad + \frac{q^2}{m_B^2-s_M}T_3(q^2)\Bigg)\Bigg]\ ,\\
h_\pm &= H_\pm(A_1\to -A_1,\ A_2\to -A_2)\ ,\\
h_S^0 &= H_S^0(A_1\to -A_1,\ A_2\to -A_2)\ ,
\end{aligned}
\tag{16}
$$

where θ is the polar angle of positron in the rest frame of the intermediate boson with respect to its helicity axis. Note that we take $p_V^2 = s_M$, but not m_V^2, in order to take into account V's being a virtual particle which subsequently decays into $\pi^+\pi^-$ or $K^-\pi^+$ pair. Remembering that the existing CLEO result [19] for the $B \to X_s \gamma$ and $B \to K^* \gamma$ decays impose strong constraint on the parameter space C_{BR} and C_{SL}. For this reason here we assume that they are equal to each other in the SM. Hence we will take

$$\begin{aligned} C_{LL}^{tot} &= C_9^{eff} - C_{10} + C_{LL} ~, \\ C_{LR}^{tot} &= C_9^{eff} + C_{10} + C_{LR} ~. \end{aligned}$$

Using the expressions of the helicity amplitudes for the differential decay rate width of the $B \to V(\to PP')\ell^+\ell^-$ decay, we get

$$\begin{aligned} d\Gamma &= \frac{3G^2\alpha^2}{2^{17}\pi^6 m_B^3 s_M q^2} \left| V_{tb} V_{tf}^* \right|^2 ds_M \, dq^2 \, d\cos\theta_P \, d\cos\theta \, d\varphi \\ &\times \lambda^{1/2}(m_B^2, s_M, q^2) \lambda^{1/2}(s_M, m_P^2, m_{P'}^2) \lambda^{1/2}(q^2, m_\ell^2, m_\ell^2) \frac{m_V \Gamma_V/\pi}{(s_M - m_V^2)^2 + m_V^2 \Gamma_V^2} \mathcal{B}(V \to PP') \\ &\times \Bigg\{ 2\cos^2\theta_P \Big[\cos^2\theta \, N_1 + \sin^2\theta \, N_2 + 2\cos\theta \, \mathrm{Re}[N_3] + N_4 \Big] \\ &+ \sin^2\theta_P \Big[\sin^2\theta \, N_5 + \Big(1+\cos^2\theta\Big) N_6 + 2\cos\theta \, N_7 + 2\sin(2\varphi)\sin^2\theta \, \mathrm{Im}[N_8] \\ &- 2\cos(2\varphi)\sin^2\theta \, \mathrm{Re}[N_8] \Big] + \sqrt{2}\sin(2\theta_P) \, \sin\theta \, \cos\varphi \, \mathrm{Re}[\cos\theta \, N_9 + N_{10}] \\ &- \sqrt{2}\sin(2\theta_P) \, \sin\theta \, \sin\varphi \, \mathrm{Im}[\cos\theta \, N_{11} + N_{12}] \Bigg\} ~, \end{aligned} \tag{17}$$

where

$$\begin{aligned} N_1 &= \left| A_0^{++} \right|^2 + \left| A_0^{--} \right|^2 ~, \\ N_2 &= \left| A_0^{+-} \right|^2 + \left| A_0^{-+} \right|^2 ~, \\ N_3 &= A_0^{++} \left(B_0^{++} \right)^* + A_0^{--} \left(B_0^{--} \right)^* ~, \\ N_4 &= \left| B_0^{++} \right|^2 + \left| B_0^{--} \right|^2 ~, \\ N_5 &= \left| A_+^{++} \right|^2 + \left| A_-^{++} \right|^2 + \left| A_+^{--} \right|^2 + \left| A_-^{--} \right|^2 ~, \\ N_6 &= \left| A_+^{+-} \right|^2 + \left| A_-^{-+} \right|^2 + \left| A_-^{+-} \right|^2 + \left| A_+^{-+} \right|^2 ~, \\ N_7 &= \left| A_-^{+-} \right|^2 + \left| A_+^{-+} \right|^2 - \left| A_+^{+-} \right|^2 - \left| A_-^{-+} \right|^2 ~, \\ N_8 &= A_+^{++} \left(A_-^{++} \right)^* + A_+^{+-} \left(A_-^{+-} \right)^* + A_+^{-+} \left(A_-^{-+} \right)^* + A_+^{--} \left(A_-^{--} \right)^* ~, \\ N_9 &= A_0^{++} \left(A_-^{++} - A_+^{++} \right)^* - A_0^{+-} \left(A_-^{+-} + A_+^{+-} \right)^* - A_0^{-+} \left(A_-^{-+} + A_+^{-+} \right)^* \\ &\quad + A_0^{--} \left(A_-^{--} - A_+^{--} \right)^* ~, \\ N_{10} &= B_0^{++} \left(A_-^{++} - A_+^{++} \right)^* + A_0^{+-} \left(-A_-^{+-} + A_+^{+-} \right)^* + A_0^{-+} \left(A_-^{-+} - A_+^{-+} \right)^* \\ &\quad + B_0^{--} \left(A_-^{--} - A_+^{--} \right)^* ~, \\ N_{11} &= N_9 \left(A_+^{++} \to -A_+^{++}, ~ A_+^{+-} \to -A_+^{+-}, ~ A_+^{-+} \to -A_+^{-+}, ~ A_+^{--} \to -A_+^{--} \right) ~, \\ N_{12} &= N_{10} \left(A_+^{++} \to -A_+^{++}, ~ A_+^{+-} \to -A_+^{+-}, ~ A_+^{-+} \to -A_+^{-+}, ~ A_+^{--} \to -A_+^{--} \right) ~, \end{aligned} \tag{18}$$

where θ_P is the polar angle of the pseudoscalar P meson momentum in the rest frame of the vector meson, with respect to the helicity axis, i.e., the outgoing direction of V meson, and φ is the azimuthal angle between the planes of the two decays $V \to PP'$ and $V^* \to \ell^+\ell^-$. Kinematically allowed region of the variables are given as

$$\begin{aligned}
(m_P + m_{P'})^2 \le s_M \le (m_B - 2m_\ell)^2 \;, \\
4m_\ell^2 \le q^2 \le (m_B - \sqrt{s_M})^2 \;, \\
-1 \le \cos\theta_P \le 1 \;, \\
-1 \le \cos\theta \le 1 \;, \\
0 \le \varphi \le 2\pi \;.
\end{aligned} \tag{19}$$

We note that, in further analysis the narrow–width approximation for V meson will be used, i.e.,

$$\lim_{\Gamma_V \to 0} \frac{\Gamma_V m_V/\pi}{(s_M - m_V^2)^2 + m_V^2\Gamma_V^2} = \delta(s_M - m_V^2) \;,$$

by means of which integration of Eq. (17) over s_M can easily be carried and the differential decay rate with respect to dilepton mass q^2, azimuthal angle φ, polar angles θ_P and θ can be written as

$$\begin{aligned}
d\Gamma &= \frac{3G^2\alpha^2}{2^{17}\pi^6 m_B^3 m_V^2 q^2} \left|V_{tb}V_{tf}^*\right|^2 \mathcal{B}(V \to PP')\, dq^2\, d\cos\theta_P\, d\cos\theta\, d\varphi \\
&\times \lambda^{1/2}(m_B^2, m_V^2, q^2)\lambda^{1/2}(m_V^2, m_P^2, m_{P'}^2)\lambda^{1/2}(q^2, m_\ell^2, m_\ell^2) \\
&\times \Bigg\{ 2\cos^2\theta_P \Big[\cos^2\theta\, N_1 + \sin^2\theta\, N_2 + 2\cos\theta\, \mathrm{Re}[N_3] + N_4 \Big] \\
&+ \sin^2\theta_P \Big[\sin^2\theta\, N_5 + \Big(1 + \cos^2\theta\Big) N_6 + 2\cos\theta\, N_7 + 2\sin(2\varphi)\sin^2\theta\, \mathrm{Im}[N_8] \\
&- 2\cos(2\varphi)\sin^2\theta\, \mathrm{Re}[N_8] \Big] + \sqrt{2}\sin(2\theta_P)\, \sin\theta\, \cos\varphi\, \mathrm{Re}[\cos\theta\, N_9 + N_{10}] \\
&- \sqrt{2}\sin(2\theta_P)\, \sin\theta\, \sin\varphi\, \mathrm{Im}[\cos\theta\, N_{11} + N_{12}] \Bigg\} \;,
\end{aligned} \tag{20}$$

for which we will use the experimental results for the $V \to PP'$ namely, $\mathcal{B}(\rho \to \pi^+\pi^-) = \mathcal{B}(K^* \to K\pi) = 1$. It should be noted here that in addition to the variables that exist in $B \to V\ell^+\ell^-$ decay, there appears a new variable θ_P, and contrary to the $B \to V\ell^+\ell^-$ case, the dependence of the cascade decay $B \to V(\to PP')\ell^+\ell^-$ on the azimuthal angle φ is not trivial. Incidentally, we would like to remind the reader that, if Eq. (20) is integrated over θ_P and φ, the differential decay rate for the $B \to V\ell^+\ell^-$ decay is obtained. The model independent analysis of the $B \to K^*\ell^+\ell^-$ decay is presented in [12], in which the dependence of the experimentally measured quantities, such as branching ratio, forward–backward asymmetry and longitudinal polarization of the final lepton and the ratio Γ_L/Γ_T of the decay widths when K^* meson is longitudinally and transversally polarized, on the new Wilson coefficients, are systematically studied. As has been noted already, our main goal in this work is to investigate the dependence of such angular distributions on the new Wilson coefficients in the $B \to V(V \to PP')\ell^+\ell^-$ decay, which does not exist in the $B \to V\ell^+\ell^-$ decay and not studied in [12].

It can easily be seen from Eq. (20) that the cascade decay $B \to V(\to PP')\ell^+\ell^-$ has a rich angular structure. Therefore, in the light of this observation, a thorough investigation

of different distributions will prove useful in separating various angular coefficients experimentally. Along the lines as suggested by [18], we adopt two different strategies in further analysis of the problem under consideration, namely investigation of the various individual angular distributions and asymmetries and their relation to the new Wilson coefficients.

For the purpose of studying single angle distributions, we integrate Eq. (20) over q^2, θ and φ, which takes the form

$$\frac{d\Gamma}{d\cos\theta_P} \sim 2\cos^2\theta_P \left[\frac{2}{3}\widetilde{N}_1 + \frac{4}{3}\widetilde{N}_2 + 2\widetilde{N}_4\right] + \sin^2\theta_P \left[\frac{4}{3}\widetilde{N}_5 + +\frac{8}{3}\widetilde{N}_6\right] , \tag{21}$$

where we introduce the notation $\widetilde{N}_i = \int N_i \, dq^2$. Defining an asymmetry parameter α_{θ_P}, from the angular distribution $W(\cos\theta_P) = 1 + \alpha_{\theta_P}\cos^2\theta_P$ we get

$$\alpha_{\theta_P} = \frac{\widetilde{N}_1 + 2\widetilde{N}_2 + 3\widetilde{N}_4}{\widetilde{N}_5 + 2\widetilde{N}_6} - 1 \; . \tag{22}$$

Integrating Eq. (20) over q^2, θ_P and φ, for the polar $\cos\theta$ distribution we get

$$\begin{aligned} \frac{d\Gamma}{d\cos\theta} \sim \; & \frac{4}{3}\Big[\cos^2\theta\,\widetilde{N}_1 + \sin^2\theta\,\widetilde{N}_2 + 2\cos\theta\,\mathrm{Re}[\widetilde{N}_3] + \widetilde{N}_4\Big] \\ & + \frac{4}{3}\Big[\sin^2\theta\,\widetilde{N}_5 + \Big(1+\cos^2\theta\Big)\widetilde{N}_6 + 2\cos\theta\,\widetilde{N}_7\Big] , \end{aligned}$$

from which one can write the angular distribution $W(\cos\theta) = 1 + \alpha_\theta\cos\theta + \beta_\theta\cos^2\theta$, with the asymmetry parameters α_θ and β_θ being defined as

$$\begin{aligned} \alpha_\theta &= \frac{2\mathrm{Re}[\widetilde{N}_3] + 2\widetilde{N}_7}{\widetilde{N}_2 + \widetilde{N}_4 + \widetilde{N}_5 + \widetilde{N}_6} , \\ \beta_\theta &= \frac{\widetilde{N}_1 - \widetilde{N}_2 - \widetilde{N}_5 + \widetilde{N}_6}{\widetilde{N}_2 + \widetilde{N}_4 + \widetilde{N}_5 + \widetilde{N}_6} . \end{aligned} \tag{23}$$

Finally we consider the azimuthal angle φ distribution, which is obtained by integrating Eq. (20) over the parameters q^2, θ_P and θ to yield

$$\begin{aligned} \frac{d\Gamma}{d\varphi} \sim \; & \frac{4}{3}\left[\frac{2}{3}\widetilde{N}_1 + \frac{4}{3}\widetilde{N}_2 + 2\widetilde{N}_4\right] \\ & + \frac{4}{3}\left[\frac{4}{3}\widetilde{N}_5 + \frac{8}{3}\widetilde{N}_6 + \frac{8}{3}\sin(2\varphi)\,\mathrm{Im}[\widetilde{N}_9] - \frac{8}{3}\cos(2\varphi)\,\mathrm{Re}[\widetilde{N}_9]\right] , \end{aligned}$$

and the azimuthal angle φ asymmetry parameters α_φ and α_φ can be extracted from $W(\varphi) = 1 + \alpha_\varphi\sin(2\varphi) + \beta_\varphi\cos(2\varphi)$ to give

$$\begin{aligned} \alpha_\varphi &= \frac{4\mathrm{Im}[\widetilde{N}_9]}{\widetilde{N}_1 + 2\widetilde{N}_2 + 3\widetilde{N}_4 + 2\widetilde{N}_5 + 4\widetilde{N}_6} , \\ \beta_\varphi &= \frac{-4\mathrm{Re}[\widetilde{N}_9]}{\widetilde{N}_1 + 2\widetilde{N}_2 + 3\widetilde{N}_4 + 2\widetilde{N}_5 + 4\widetilde{N}_6} . \end{aligned} \tag{24}$$

A second strategy for separating various angular coefficients experimentally, is to define suitable asymmetry ratios that project out the partial rates from Eq. (20). For this purpose we consider the following asymmetries (see also [18])

$$\begin{aligned} A_\varphi &= \frac{d\Gamma(\varphi) - d\Gamma(\varphi+\pi/2) + d\Gamma(\varphi+\pi) - d\Gamma(\varphi+3\pi/2)}{d\Gamma(\varphi) + d\Gamma(\varphi+\pi/2) + d\Gamma(\varphi+\pi) + d\Gamma(\varphi+3\pi/2)}\,, \qquad (25)\\ & -\frac{\pi}{4} \le \varphi \le \frac{\pi}{4}\,, \\ A_1 &= \frac{N}{D}\,, \qquad (26)\end{aligned}$$

where

$$\begin{aligned} N &= d\Gamma(\theta,\theta_P,\varphi) - d\Gamma(\theta,\theta_P,\varphi+\pi) - d\Gamma(\theta,\pi-\theta_P,\varphi) + d\Gamma(\theta,\pi-\theta_P,\varphi+\pi)\\ &- d\Gamma(\pi-\theta,\theta_P,\varphi) + d\Gamma(\pi-\theta,\theta_P,\varphi+\pi) + d\Gamma(\pi-\theta,\pi-\theta_P,\varphi)\\ &- d\Gamma(\pi-\theta,\pi-\theta_P,\varphi+\pi)\,,\end{aligned}$$

$$\begin{aligned} & 0 \le \theta_P \le \pi/2\,,\\ & \frac{\pi}{2} \le \theta \le \pi\,,\\ & -\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}\,,\end{aligned}$$

and the denominator D is given by the same expression, with plus signs everywhere,

$$A_2 = \frac{d\Gamma(\theta_P,\varphi) - d\Gamma(\theta_P,\varphi+\pi) - d\Gamma(\pi-\theta_P,\varphi) + d\Gamma(\pi-\theta_P,\varphi+\pi)}{d\Gamma(\theta_P,\varphi) + d\Gamma(\theta_P,\varphi+\pi) + d\Gamma(\pi-\theta_P,\varphi) + d\Gamma(\pi-\theta_P,\varphi+\pi)}\,, \qquad (27)$$

$$\begin{aligned} & 0 \le \theta_P \le \pi/2\,,\\ & -\frac{\pi}{2} \le \varphi \le \pi/2\,.\end{aligned}$$

In expressions (25)–(27) the angles that do not appear in the arguments of the differential rate $d\Gamma$ have been integrated out over their physical ranges ($0 \le \theta,\ \theta_P \le \pi,\ 0 \le \varphi \le 2\pi$). Integrating over all variables, we are left with the expressions $\widetilde{A}_\varphi,\ \widetilde{A}_1,\ \widetilde{A}_2$, which depend only on the Wilson coefficients, as follows (here $\widetilde{\ }$ in the notation refers to integration's being performed over all variables)

$$\widetilde{A}_\varphi = \frac{-8\mathrm{Re}[\widetilde{N}_9]}{\pi\left(\widetilde{N}_1 + 2\widetilde{N}_2 + 3\widetilde{N}_4 + 2\widetilde{N}_5 + 4\widetilde{N}_6\right)}\,, \qquad (28)$$

$$\widetilde{A}_1 = \frac{-\sqrt{2}\mathrm{Re}[\widetilde{N}_{10}]}{\pi\left(\widetilde{N}_1 + 2\widetilde{N}_2 + 3\widetilde{N}_4 + 2\widetilde{N}_5 + 4\widetilde{N}_6\right)}\,, \qquad (29)$$

$$\widetilde{A}_2 = \frac{3\mathrm{Re}[\widetilde{N}_{11}]}{\sqrt{2}\left(\widetilde{N}_1 + 2\widetilde{N}_2 + 3\widetilde{N}_4 + 2\widetilde{N}_5 + 4\widetilde{N}_6\right)}\,. \qquad (30)$$

Before proceeding further, we would like to consider the CP–violating observables that can be constructed by combining the information on $\bar{B}$ and B decays, namely, $\bar{B} \to P\bar{P}'\ell^+\ell^-$

and $B \to \bar{P}P'\ell^+\ell^-$, and define CP–odd asymmetry in the following way

$$\widetilde{A}_{CP} \equiv \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} ~, \tag{31}$$

where Γ and $\bar{\Gamma}$ are the decay widths of the $\bar{B} \to P\bar{P}'\ell^+\ell^-$ and $B \to \bar{P}P'\ell^+\ell^-$ processes, respectively. Explicit form of Γ can easily be obtained from Eq. (20), by performing integration over the variables q^2, θ, θ_P, and φ. The decay width for the conjugate process can again be obtained from Eq. (20) by making the replacement $N_i \to \bar{N}_i$, where $\bar{N}_i$ are the functions for the conjugate processes. It should be noted that we consider a case in which all new Wilson coefficients are real. Furthermore, form factors which enter into Eq. (16) are computed in framework of light cone QCD sum rules method [20]–[22] and this nonperturbative approach predicts that all form factors are real as well. In other words, there is no any new source for CP violation other than that are present in the SM. As is well known, for the $B \to \rho\ell^+\ell^-$ decay in the SM only the coefficient C_9^{eff} contains both a weak phase φ_W (associated with the imaginary part of the CKM matrix element) and a strong phase δ_S (attributed to the imaginary parts of the $c\bar{c}$ and $b\bar{b}$ loops). As a result of these considerations, it follows then that the decay width for the conjugate $B \to \bar{P}P'\ell^+\ell^-$ process can be obtained from $\bar{B} \to P\bar{P}'\ell^+\ell^-$ channel by the replacements $\varphi_W \to -\varphi_W$ and $\delta_S \to \delta_S$.

3 Numerical analysis

In this section we present our numerical results for the asymmetries A_{CP}, α_{θ_P}, α_θ, β_θ, β_φ , A_φ , α_φ , $\widetilde{A}_1$, $\widetilde{A}_\varphi$ and $\widetilde{A}_2$ for the exclusive rare $B \to \pi^+\pi^-\ell^+\ell^-$ decay only. We take hadronic form factors from Table I and the Wilson coefficients from Table II. The values of the main input parameters used in our analysis are: $m_b = 4.8~GeV$, $m_c = 1.35~GeV$, $m_\rho = 0.77~GeV$, $m_\tau = 1.78~GeV$, $m_\mu = 0.105~GeV$ and $m_B = 5.28~GeV$. Here we note that the results for $B \to K\pi\ell^+\ell^-$ can be obtained from $B \to \pi\pi\ell^+\ell^-$ by replacing V_{td} with V_{ts}, $B \to \rho$ transition form factors with $B \to K$ transition form factors, and m_π with m_K.

In the present work we choose light cone QCD sum rules method predictions for the form factors. In our numerical analysis we will use the results of the work [21, 22] in which the form factors are described by a three–parameter fit where the radiative corrections up to leading twist contribution and SU(3)–breaking effects are taken into account. The q^2–dependence of the form factors, which appears in our analysis could be parametrized as

$$F(s) = \frac{F(0)}{1 - a_F\, s + b_F\, s^2} ~,$$

where $s = q^2/m_B^2$ is the dilepton invariant mass in units of B meson mass, and the parameters $F(0)$, a_F and b_F are listed in Table 1 for each form factor. In the SM the Wilson coefficients $C_7^{eff}(m_b)$ and $C_{10}(m_b)$, whose analytical expressions are given in [23, 24], are strictly real as can be read off from Table 2. In the leading logarithmic approximation, at the scale $\mathcal{O}(\mu = m_b)$, we have

$$\begin{aligned} C_7^{eff}(m_b) &= -0.313 ~, \\ C_{10}^{eff}(m_b) &= -4.669 ~. \end{aligned}$$

	$F(0)$	a_F	b_F
$A_0^{B\to\rho}$	0.372	1.40	0.437
$A_1^{B\to\rho}$	0.261	0.29	-0.415
$A_2^{B\to\rho}$	0.223	0.93	-0.092
$V^{B\to\rho}$	0.338	1.37	0.315
$T_1^{B\to\rho}$	0.143	1.41	0.361
$T_2^{B\to\rho}$	0.143	0.28	-0.500
$T_3^{B\to\rho}$	0.101	1.06	-0.076

Table 1: The form factors for $B \to \rho \ell^+ \ell^-$ in a three–parameter fit.

C_1	C_2	C_3	C_4	C_5	C_6	C_7^{eff}	C_9	C_{10}^{eff}	$C^{(0)}$
-0.248	1.107	0.011	-0.026	0.007	-0.031	-0.313	4.344	-4.669	0.362

Table 2: The numerical values of the Wilson coefficients at $\mu \sim m_b$ scale within the SM.

Although individual Wilson coefficients at $\mu \sim m_b$ level are all real (see Table 2), the effective Wilson coefficient $C_9^{eff}(m_b, \hat{s})$ has a finite phase, and in next–to–leading order

$$C_9^{eff}(m_b, \hat{s}) = C_9(m_b) \left[1 + \frac{\alpha_s(\mu)}{\pi} \omega(\hat{s})\right] + Y_{SD}(m_b, \hat{s}) + Y_{LD}(m_B, s) \ , \tag{32}$$

where $C_9(m_b) = 4.344$. Here $\omega\,(\hat{s})$ represents the $\mathcal{O}(\alpha_s)$ corrections coming from one–gluon exchange in the matrix element of the corresponding operator, whose explicit form can be found in [23]. In (32) Y_{SD} and Y_{LD} represent, respectively, the short– and long–distance contributions of the four–quark operators $\mathcal{O}_{i=1,\cdots,6}$ [23, 24]. Here Y_{SD} can be obtained by a perturbative calculation

$$\begin{aligned} Y_{SD}\,(m_b, \hat{s}) &= g\,(\hat{m}_c, \hat{s})\, C^{(0)} - \frac{1}{2} g\,(1, \hat{s})\,[4C_3 + 4C_4 + 3C_5 + C_6] \\ &- \frac{1}{2} g\,(0, \hat{s})\,[C_3 + 3C_4] + \frac{2}{9}\,[3C_3 + C_4 + 3C_5 + C_6] \\ &- \lambda_u\,[3C_1 + C_2]\,[g\,(0, \hat{s}) - g\,(\hat{m}_c, \hat{s})] \ , \end{aligned}$$

where

$$\begin{aligned} C^{(0)} &= 3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6 \ , \\ \lambda_u &= \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*} \ , \end{aligned}$$

and the loop function $g\,(m_q, s)$ stands for the loops of quarks with mass m_q at the dilepton invariant mass s. This function develops absorptive parts for dilepton energies $s = 4m_q^2$:

$$
\begin{aligned}
g\,(\hat{m}_q, \hat{s}) &= -\frac{8}{9}\ln \hat{m}_q + \frac{8}{27} + \frac{4}{9} y_q - \frac{2}{9}\,(2 + y_q)\sqrt{|1 - y_q|} \\
&\times \left[\Theta(1 - y_q)\left(\ln \frac{1 + \sqrt{1 - y_q}}{1 - \sqrt{1 - y_q}} - i\pi \right) + \Theta(y_q - 1)\, 2 \arctan \frac{1}{\sqrt{y_q - 1}} \right],
\end{aligned}
$$

where $\hat{m}_q = m_q/m_b$ and $y_q = 4\hat{m}_q^2/\hat{s}$. In addition to these perturbative contributions, the $\bar{c}c$ loops can excite low–lying charmonium states $\psi(1s), \cdots, \psi(6s)$ whose contributions are represented by Y_{LD} [25]:

$$
\begin{aligned}
Y_{LD}\,(m_b, \hat{s}) &= \frac{3}{\alpha^2}\left[-\frac{V_{cf}^* V_{cb}}{V_{tf}^* V_{tb}}\, C^{(0)} - \frac{V_{uf}^* V_{ub}}{V_{tf}^* V_{tb}}\,(3C_3 + C_4 + 3C_5 + C_6) \right] \\
&\times \sum_{V_i = \psi(1s), \cdots, \psi(6s)} \frac{\pi \kappa_i \Gamma\,(V_i \to \ell^+ \ell^-)\, M_{V_i}}{\left(M_{V_i}^2 - \hat{s} m_b^2 - i M_{V_i} \Gamma_{V_i} \right)}\ ,
\end{aligned}
$$

where κ_i are the Fudge factors (see for example [7]).

Let us first study the dependence of the asymmetry parameter α_{θ_P} on the new Wilson coefficients. Note that in further analysis, only short distance contributions are taken into account and integration over q^2 is performed in the full physical region $4m_\ell^2 \le q^2 \le (m_B - m_V^2)^2$. We assumed that all new Wilson coefficients C_X are real, i.e., we do not introduce any new phase in addition to the one present in the SM.

In Figs. (1) and (2), we present the dependence of α_{θ_P} on the new Wilson coefficients, for the $B \to \pi^+\pi^- e^+ e^-$ and $B \to \pi^+\pi^-\tau^+\tau^-$ decays, respectively. Here and in all of the following figures, zero value of new Wilson coefficients C_X correspond to the SM prediction. In the case of $B \to \pi^+\pi^- e^+ e^-$ decay the asymmetry parameter α_{θ_P} is more sensitive to C_{LL}^{tot} and C_{RL}, while for the $B \to \pi^+\pi^-\tau^+\tau^-$ decay it depends strongly on C_{RL}. These dependencies can be explained as follows. For the $B \to \pi^+\pi^- e^+ e^-$ decay, if the terms proportional to electron mass are neglected, it easily be seen from Eq. (22) that

$$
\alpha_{\theta_P} = \frac{\widetilde{N}_2}{\widetilde{N}_6} - 1\ .
$$

In the limit $v \to 1$ we get

$$
\begin{aligned}
\widetilde{N}_2 \simeq \left| q^2 \right| &\left| 2\left(C_9^{eff} + C_!0 + C_{LR}^{tot} \right) H_0 - 4C_7^{eff} \frac{m_b}{q^2} \mathcal{H}_0 + C_{RR} h_0 \right|^2 \\
+ \left| q^2 \right| &\left| 2\left(C_9^{eff} - C_!0 + C_{LL}^{tot} \right) H_0 - 4C_7^{eff} \frac{m_b}{q^2} \mathcal{H}_0 + C_{RL} h_0 \right|^2\ .
\end{aligned}
\tag{33}
$$

In the SM in the large dilepton mass region, say about $q^2 \simeq 5\ GeV^2$, $C_9^{eff} + C_{10} \simeq 0.4$ and $\mathrm{Re}\left[C_9^{eff} - C_{10} \right] \simeq 9.5$. It follows then that the interference terms between $C_9^{eff} - C_{10}$ and C_{LL} (C_{RL}) are dominant and hence contributions coming from C_{LL} (C_{RL}) are large. These figures illustrates that the contributions of C_{LL} and C_{RL} to α_{θ_P} is positive for $C_{LL} > 0$ and $C_{RL} < 0$, and negative for $C_{LL} < 0$ and $C_{RL} > 0$. It should be noted that the asymmetry

parameter α_{θ_P} can get only positive or negative values for the case $C_{LL} \neq 0$ and $C_{RL} \neq 0$, while it is always positive for all other choices of the Wilson coefficients, as is the case in the SM. For this reason determination of the sign of α_{θ_P} can serve as an efficient tool for establishing new physics.

In the $B \to \pi^+\pi^-\tau^+\tau^-$ process however, the situation is slightly different compared to that of the $B \to \pi^+\pi^-e^+e^-$ transition. Largest contribution in this case comes from C_{RL} and contributions from all other Wilson coefficients are comparable to one another. This observation can be attributed to mass of the τ lepton, for which α_{θ_P} is positive for the choice of each individual new Wilson coefficients.

Depicted in Figs. (3), (4) and Figs. (5), (6) are the dependencies of the asymmetry parameters α_θ and β_θ on the new Wilson coefficients C_X, for the $B \to \pi^+\pi^-e^+e^-$ and $B \to \pi^+\pi^-\tau^+\tau^-$ decays, respectively. Figs. (3) and (5) depict that the asymmetry parameter α_θ for the e^+e^- channel depends strongly on the new Wilson coefficient C_{RL}, while it displays similar behavior for all new Wilson coefficients for the $\tau^+\tau^-$ case. We observe that the asymmetry parameter β_θ depends strongly on C_{RL} in the e^+e^- channel and on C_{RL} and C_{RR} in the $\tau^+\tau^-$ channel. It is interesting that β_θ changes its sign when $C_{RL} > 2$ in the e^+e^- channel, while it is negative for all values of C_{RL} or C_{RR} in the $\tau^+\tau^-$ channel. Therefore determination of the sign of β_θ is useful in looking for new physics.

In Figs. (7) and (8) we present the dependence of the asymmetry parameter β_φ for the $B \to \pi^+\pi^-e^+e^-$ and $B \to \pi^+\pi^-\tau^+\tau^-$ decays, respectively. From these figures one notices that this asymmetry parameter is quite sensitive to the variation in C_{RL} and changes its sign at $C_{RR} > 2$ for both channels. Our investigation of the dependence of the asymmetry parameter α_φ on the new Wilson coefficients shows that for the range $-4 < C_X < 4$, α_φ varies between -6×10^{-3} to 6×10^{-3} for the $B \to \pi^+\pi^-e^+e^-$ and -5.0×10^{-3} to 2.5×10^{-3} for the $B \to \pi^+\pi^-\tau^+\tau^-$ decays, repectively. Therefore detection of the dependence of the asymmetry parameter α_φ on the new Wilsom coefficients is quite hard from experimental point of view.

The asymmetry parameter $\widetilde{A}_1$ shows strong dependence on C_{RL} and C_{LL} for the e^+e^- channel, as depicted in Fig. (9), whose contributions are dominant compared to the other Wilson coefficients. For the $\tau^+\tau^-$ channel contributions of C_{RL} and C_{RR} become dominant, as can be seen in Fig. (10). The asymmetry parameter $\widetilde{A}_2$ varies considerably for the e^+e^- channel, in relation to the variations occurring in C_{RL}, while this behavior is switched to the Wilson coefficient C_{LRRL} for the $\tau^+\tau^-$ case which are presented in Figs. (11) and (12).

Presented in Figs. (13) and (14) are the dependence of the asymmetry parameter A_φ on new Wilson coefficients for the $e^+e^-\pi^+\pi^-$ and $\tau^+\tau^-\pi^+\pi^-$ decays, respectively. In both cases the asymmetry parameter A_φ shows strong dependence on C_{RL}.

Finally, in Figs. (15) and (16) we present the dependence of the averaged (i.e., integrated over q^2 in the full physical region) CP asymmetry on the Wilson coefficients for the $B \to \pi^+\pi^-e^+e^-$ and $B \to \pi^+\pi^-\tau^+\tau^-$ decays, respectively. We observe that $\langle A_{CP}\rangle$ is strongly dependent on C_{RL} and C_{RR} for the e^+e^- channel, while this dependence is switched to C_{LL} for the $\tau^+\tau^-$ channel. One can easily read from this figures that $\langle A_{CP}\rangle > -2.0 \times 10^{-2}$ in the e^+e^- channel, for the values $C_{RR} \leq -2$ and $|\langle A_{CP}\rangle| > 1.7 \times 10^{-2}$ when $C_{LL} \leq -2$.

As the final concluding remark, we presented in this work the model independent analysis of the exclusive $B \to \pi^+\pi^-\ell^+\ell^-$ ($\ell = e,\ \tau$) decay is presented. In particular, the sensitivity to the new Wilson coefficients of the experimentally measurable asymmetries and CP violating asymmetry are systematically analyzed. The main result of the present study is that different asymmetry parameters show strong dependence on different new

Wilson coefficients. Therefore a combined analysis of the different asymmetries and CP violating asymmetry can give unambiguous information about the existence of new physics beyond the SM and especially about various new Wilson coefficients.

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Figure captions

Fig. 1 The dependence of the asymmetry parameter α_{θ_P} on the new Wilson coefficients for the $B \to \pi^+\pi^- e^+ e^-$ decay.

Fig. 2 The same as in Fig. (1), but for the $B \to \pi^+\pi^-\tau^+\tau^-$ decay.

Fig. 3 The same as in Fig. (1), but for the asymmetry parameter α_θ.

Fig. 4 The same as in Fig. (1), but for the asymmetry parameter β_θ.

Fig. 5 The same as in Fig. (3), but for the $B \to \pi^+\pi^-\tau^+\tau^-$ decay.

Fig. 6 The same as in Fig. (4), but for the $B \to \pi^+\pi^-\tau^+\tau^-$ decay.

Fig. 7 The same as in Fig. (1), but for the asymmetry parameter β_φ.

Fig. 8 The same as in Fig. (7), but for the $B \to \pi^+\pi^-\tau^+\tau^-$ decay.

Fig. 9 The same as in Fig. (1), but for the asymmetry parameter $\widetilde{A}_1$.

Fig. 10 The same as in Fig. (9), but for the $B \to \pi^+\pi^-\tau^+\tau^-$ decay.

Fig. 11 The same as in Fig. (9), but for the asymmetry parameter $\widetilde{A}_2$.

Fig. 12 The same as in Fig. (11), but for the $B \to \pi^+\pi^-\tau^+\tau^-$ decay.

Fig. 13 The same as in Fig. (9), but for the asymmetry parameter $\widetilde{A}_\varphi$.

Fig. 14 The same as in Fig. (13), but for the $B \to \pi^+\pi^-\tau^+\tau^-$ decay.

Fig. 15 The dependence of the averaged CP asymmetry on the new Wilson coefficients for the $B \to \pi^+\pi^- e^+ e^-$ decay.

Fig. 16 The same as in Fig. (15), but for the $B \to \pi^+\pi^-\tau^+\tau^-$ decay.

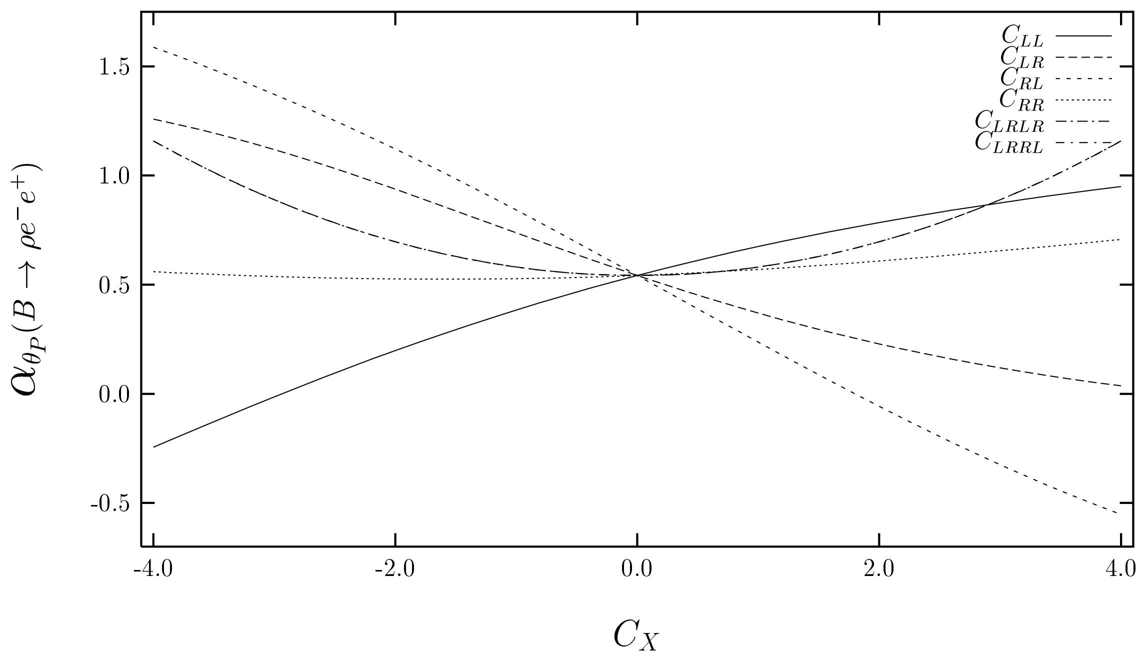
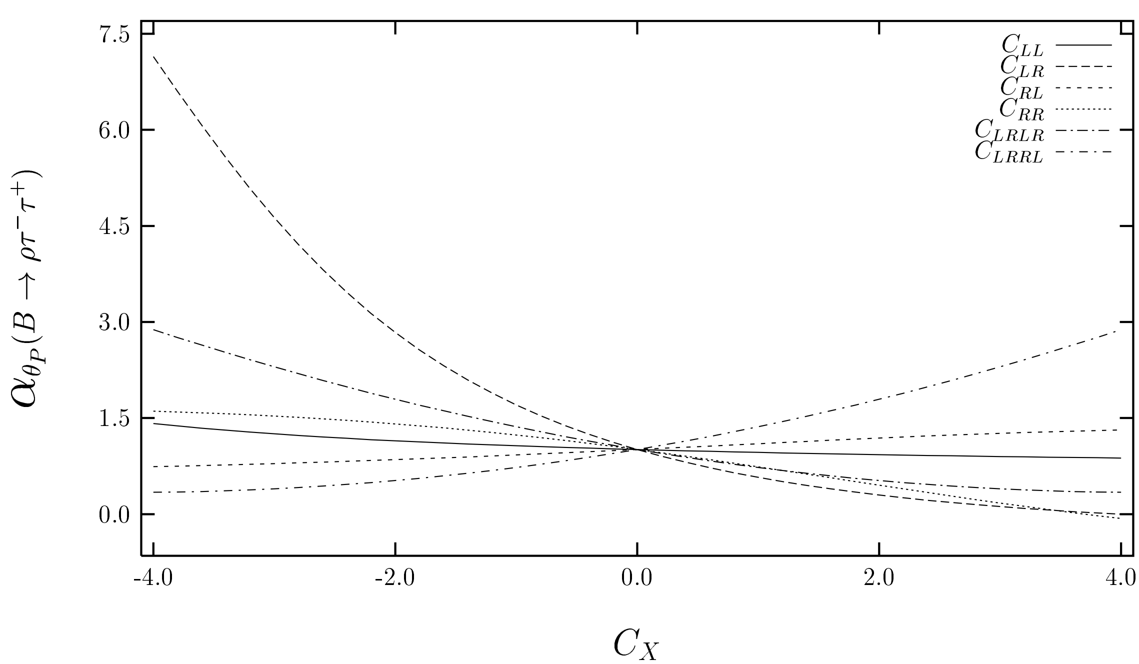


Figure 1:



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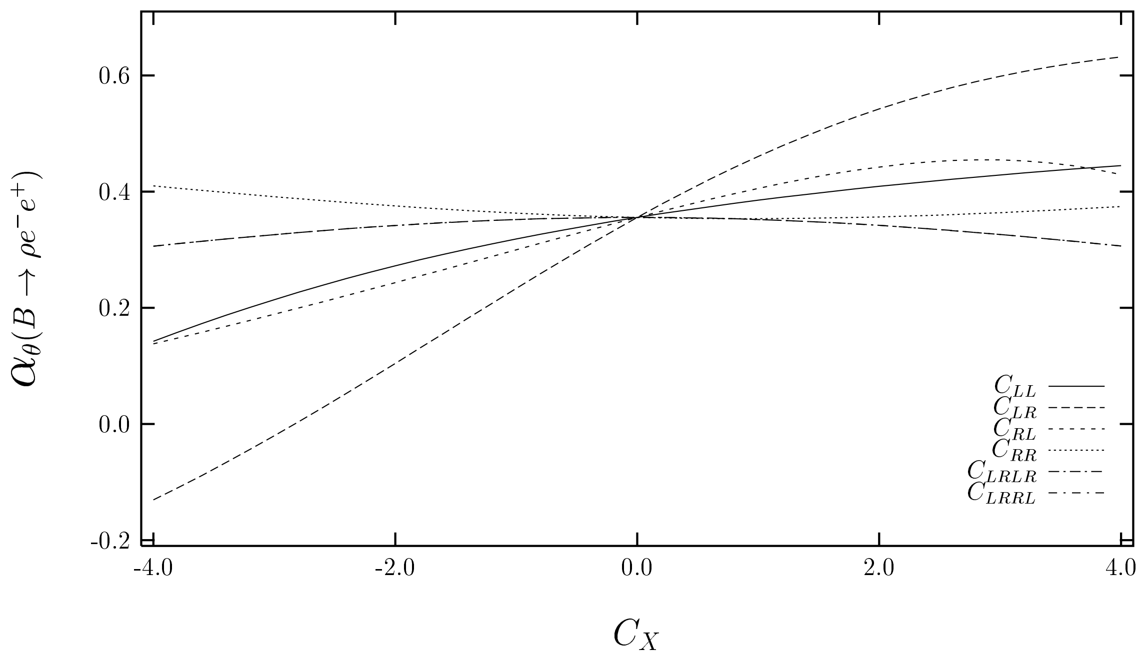

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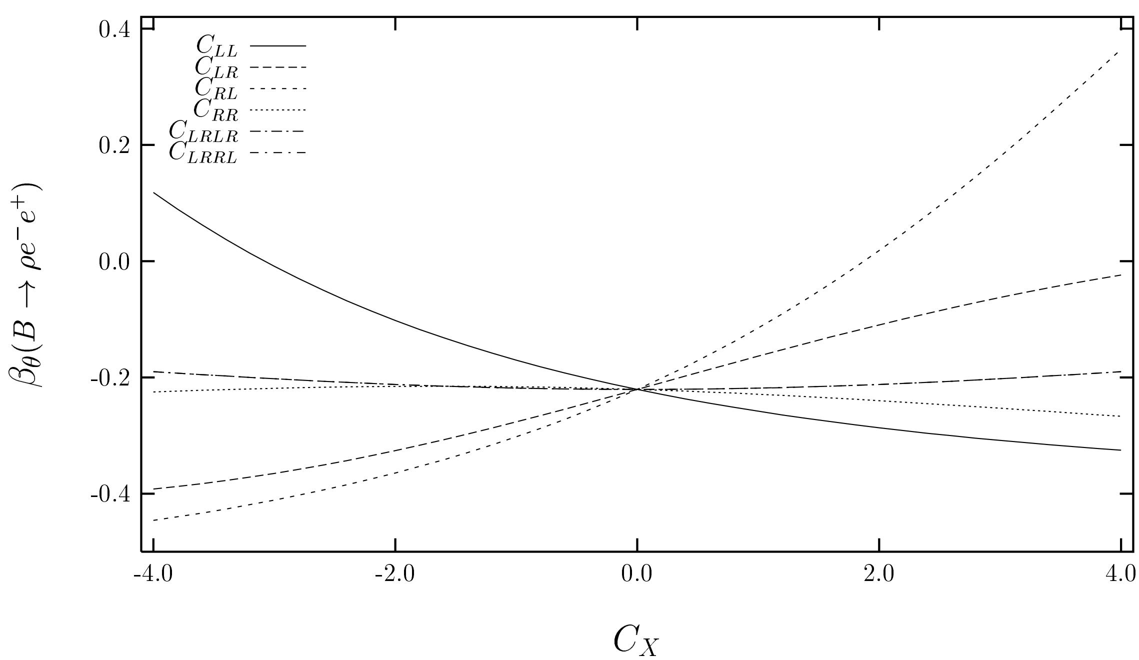

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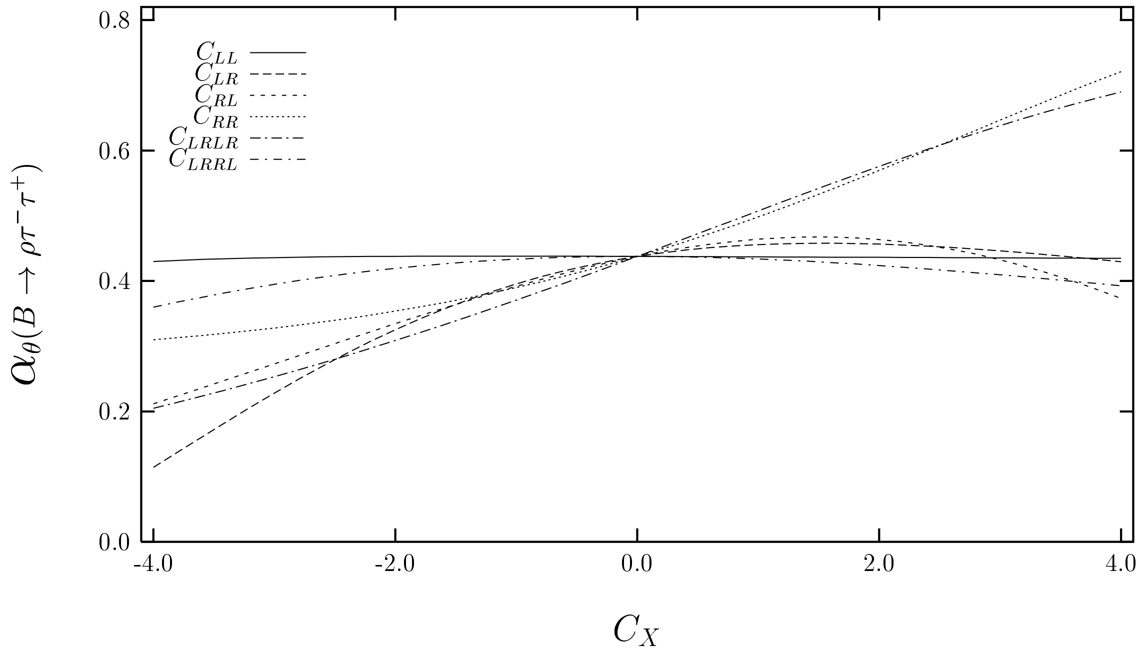

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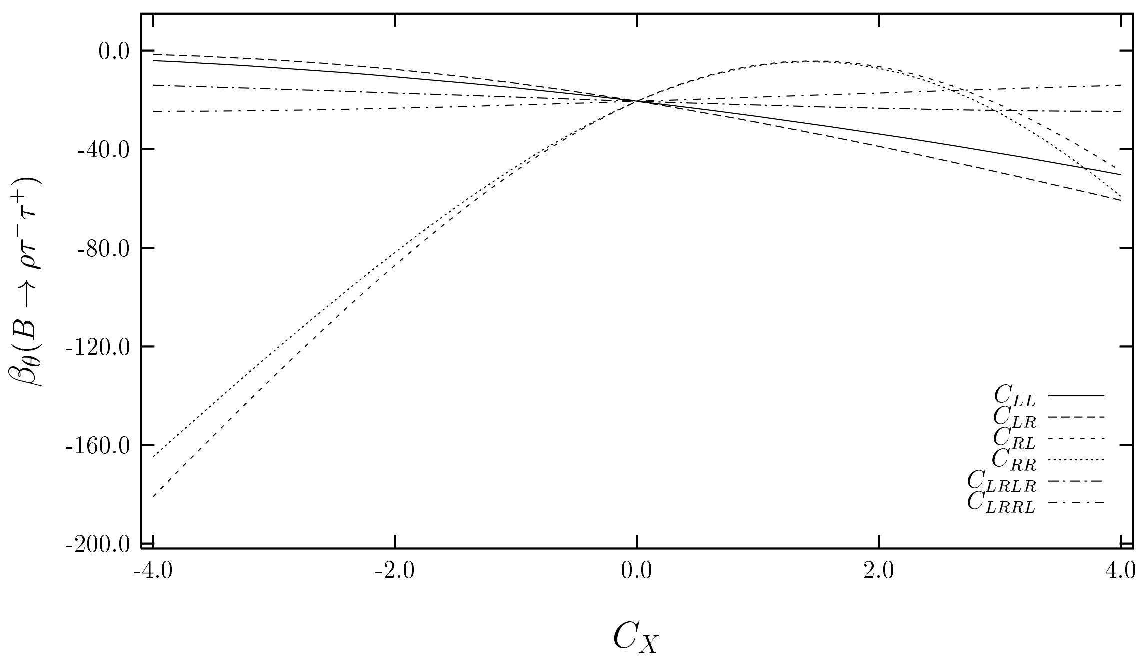

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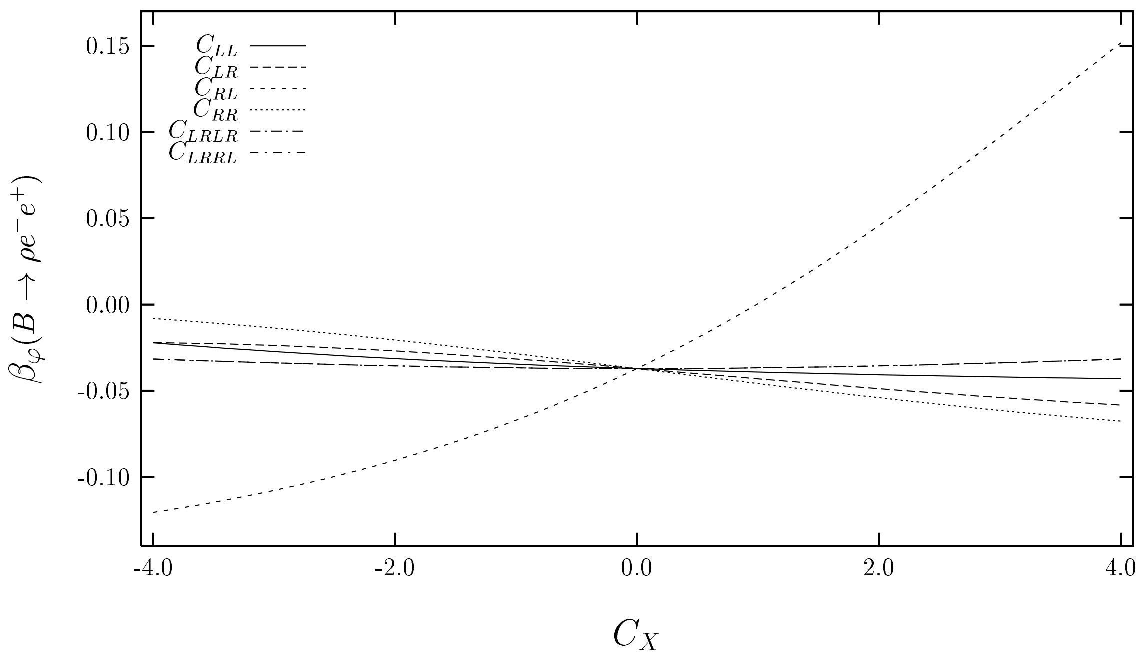

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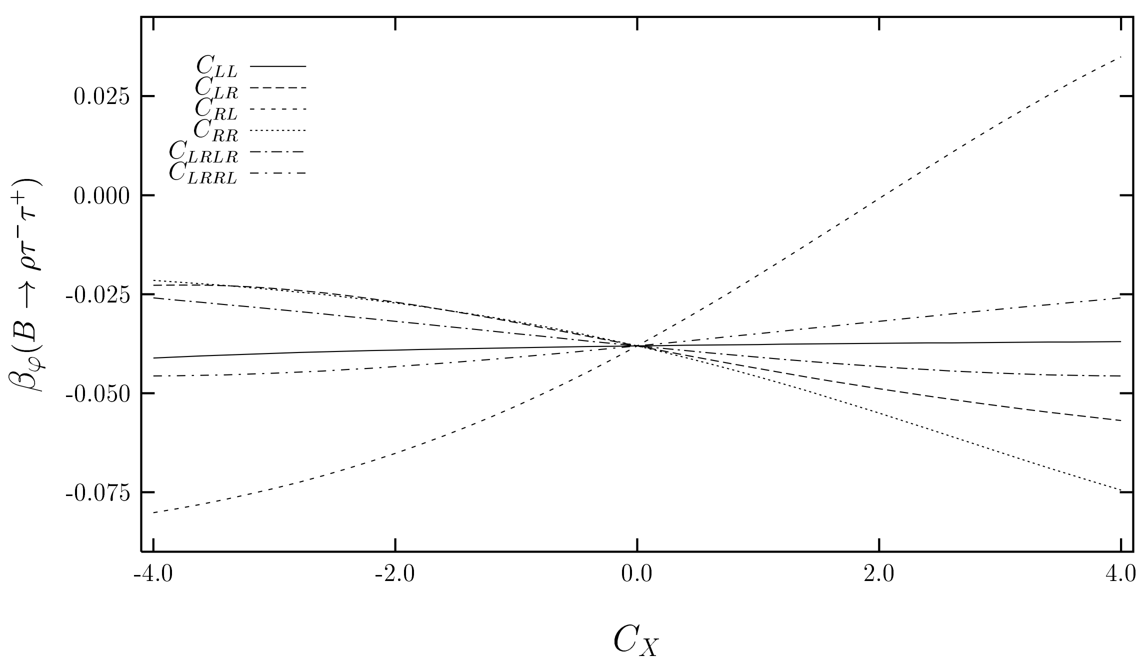

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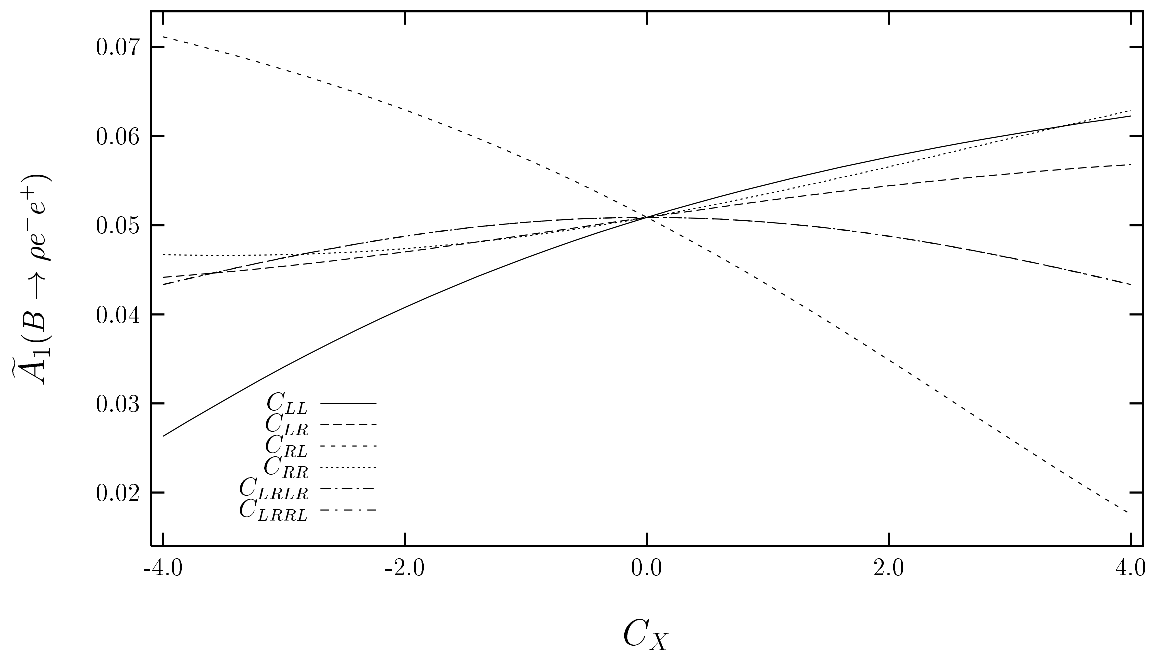

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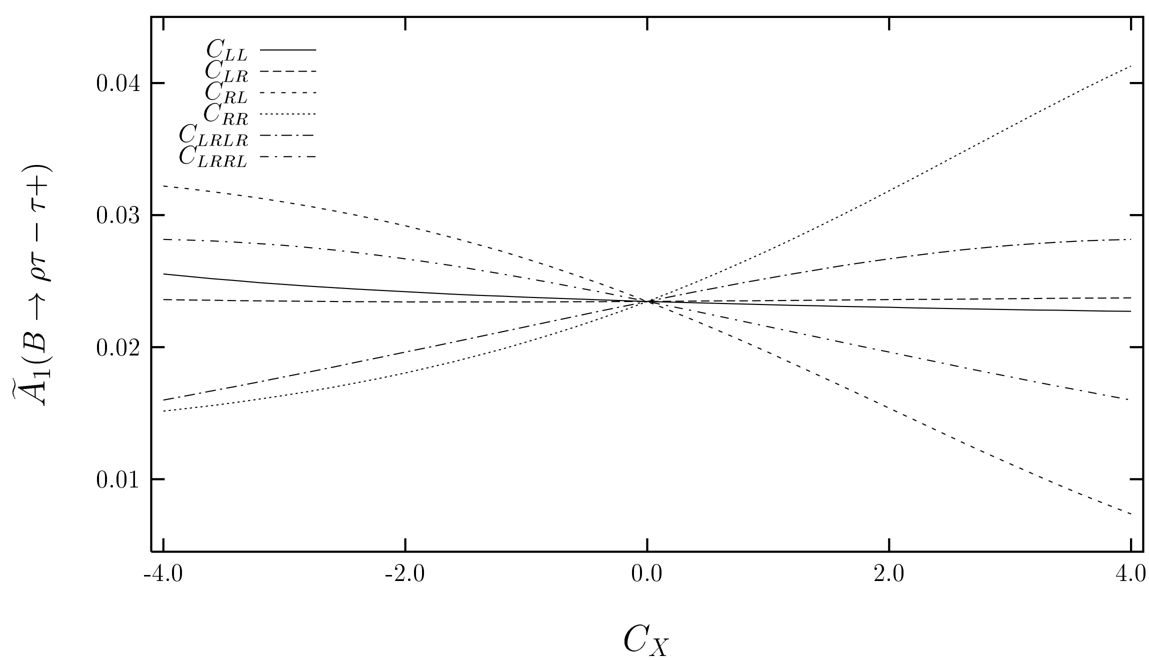
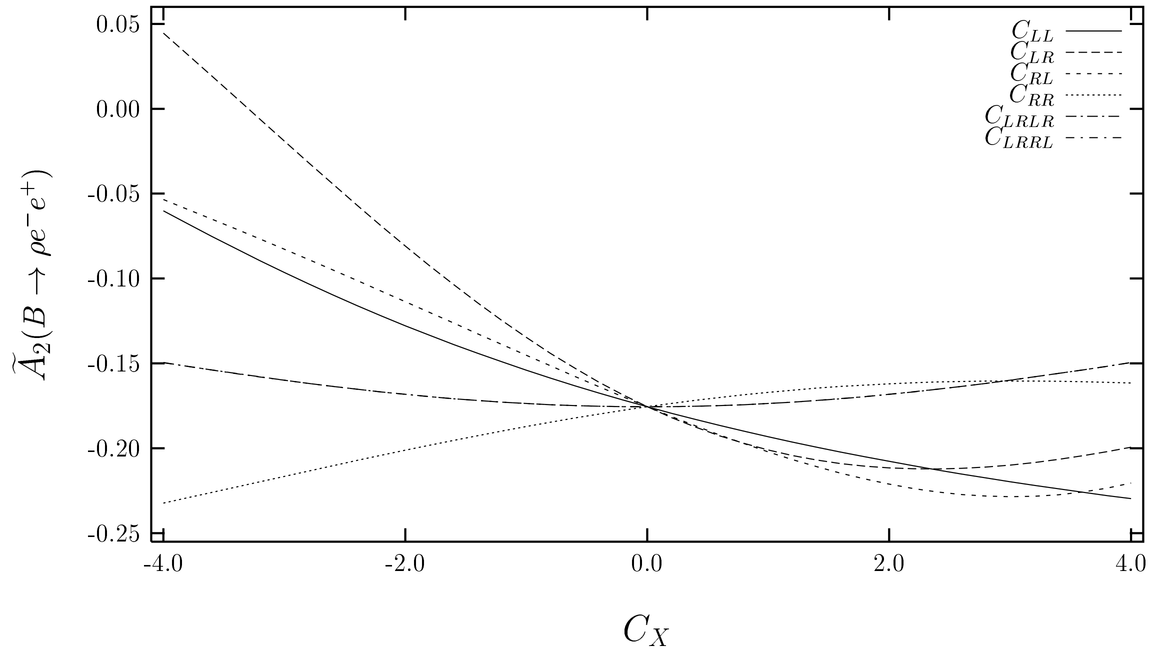


Figure 10:



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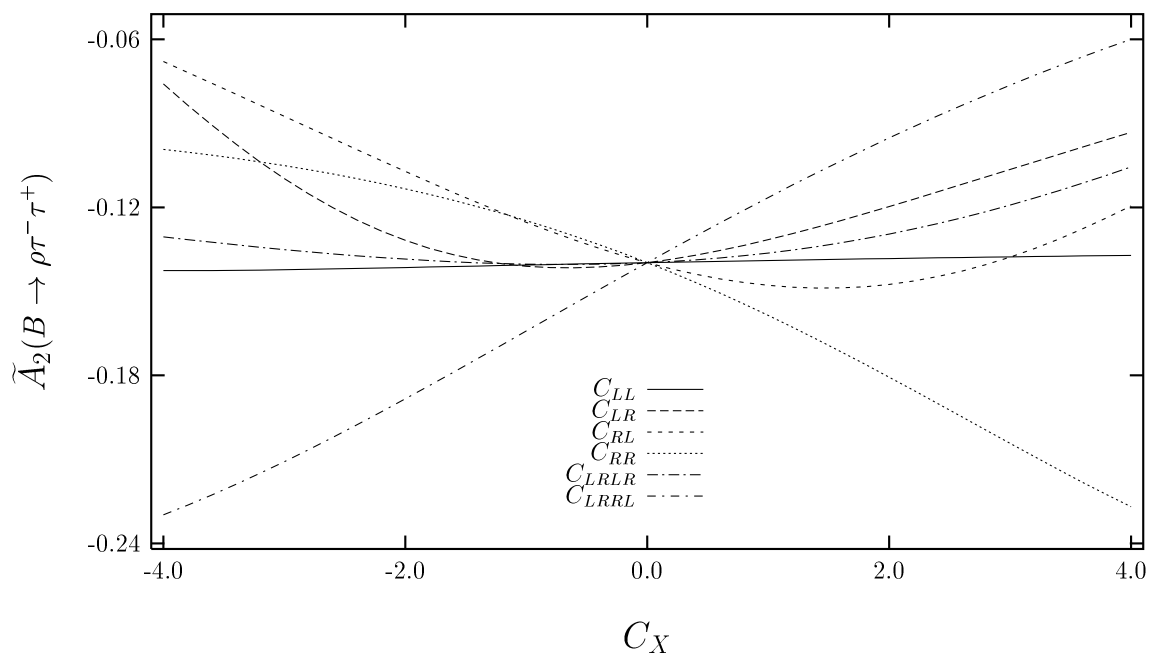

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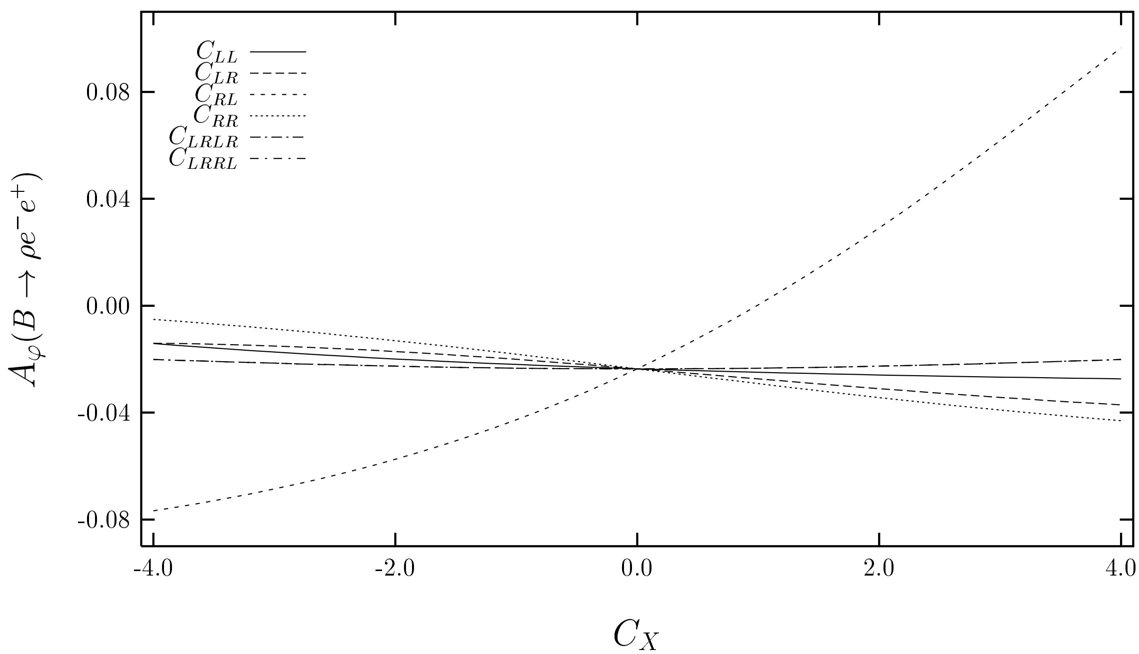

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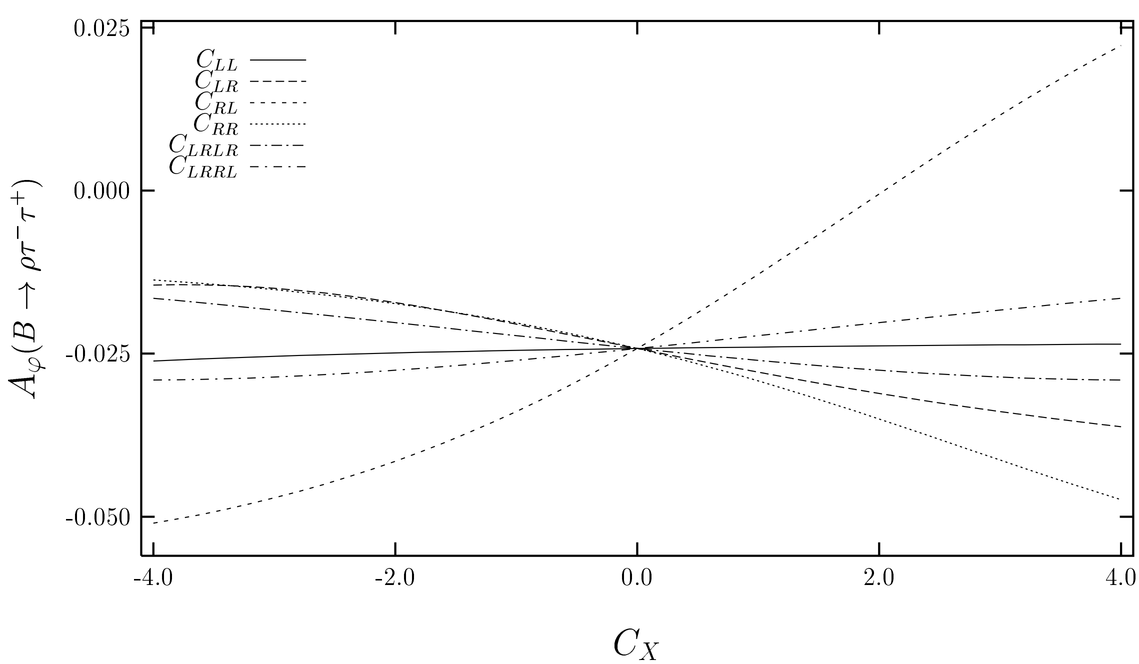

Figure 14:

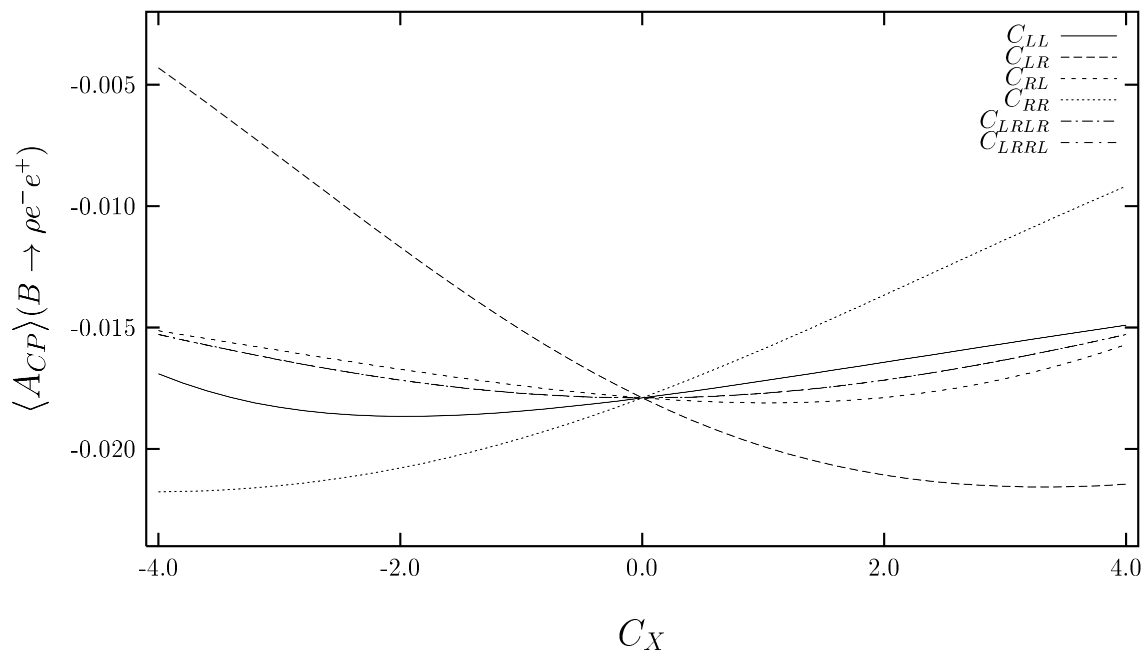

Figure 15:

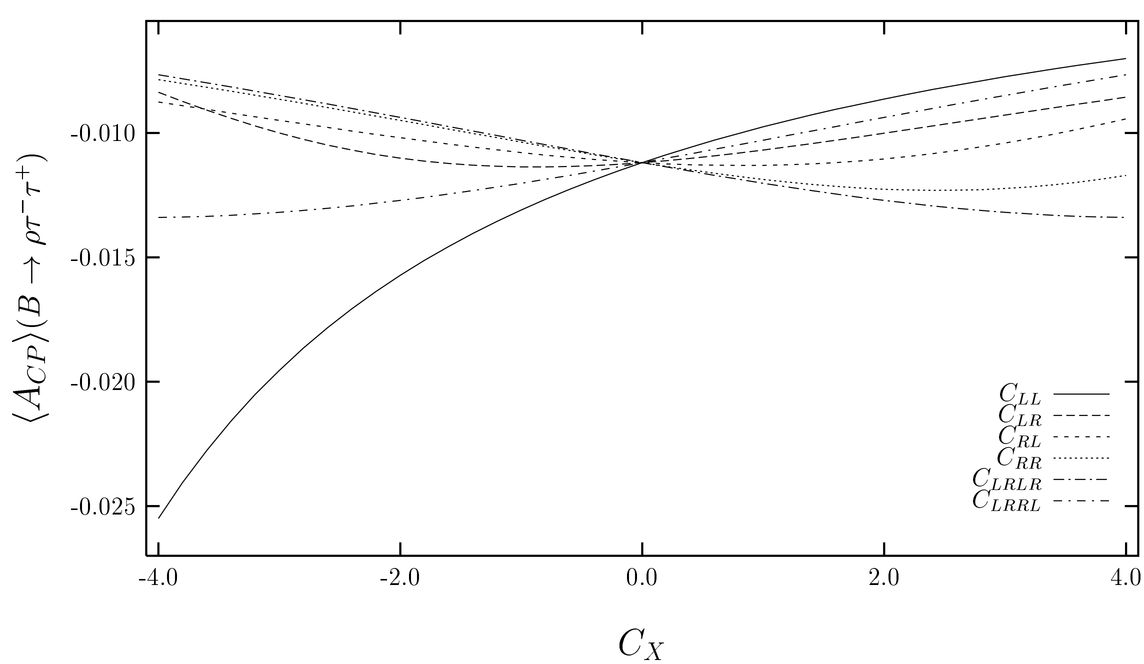

Figure 16: